A Dimensional Theory of Social Diffusion: An Analysis, Modeling, and Partial Testing of One-Way Interacting

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A DIMENSIONAL THEORY OF SOCIAL DIFFUSION

An Analysis, Modeling, and Partial Testing of One-Way Interacting

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I. THE FIELD OF DIFFUSION THEORY

This theory of social diffusion attempts to deal mathematically with one of the basic problems of social psychology and sociology, namely: How does novel behavior, once it occurs in at least one person, diffuse through a given population, particularly when its chief means of spread is by person-to-person interaction? Novel behavior means a response or activity, including the learning of information, which has not appeared before in that population in the current specific form and which may exist in any sensory modality. Behavior diffuses (or "is imitated" we often say) under various types of social constraint, with resulting diverse forms of distribution of the acts in time and distribution of the acts among people. Each of these constraints is translatable into quantitative terms.

Diffusion has been an area of interest to anthropologists studying the spread of culture. How does a new culture trait grow in time through a homogeneous population starting with its innovators and ending up as a trait of everyone? A theory of diffusion charts the growth curves of culture. Diffusion has been an area of special interest to sociometrists studying the development of networks (15). How does an interperson network of friendships arise, grow, or decline? How do opinions and rumors spread? A theory of diffusion should show the mechanism, explaining such sociometric networks of communication. From this has developed an interest in social psychology for studying rumor or the communicating of any news item. How fast does a rumor spread under specifiable conditions? A theory of diffusion should predict the temporal course of a rumor or other news item as it converts non-knowers into knowers.

Any scientific theory of that kind of novel social behavior which diffuses on an atomistic or person-to-person basis, if it is to serve the predicting function of science, must fulfill two basic objectives. (1) The theory must develop theoretical models which weave any number of the many types of conditions or constraints which govern diffusion into a homogeneous mathematical warp and woof. These conditions will often be expressed mathematically as parameters in the equations, i.e., as constants related to variables.

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The models must thus be internally consistent, systematizing new knowledge with current knowledge. (2) The theory must comprise models whose concepts can be operationally defined and whose parameters and variables can be determined by ordinary statistical procedures. The models must thus correspond to external reality, yielding close and good fits to observed data. These two objectives are the time-honored tests of truth in science—the consistency test using deduction largely and the correspondence-to-phenomena test using induction largely.

Our theory of social diffusion can be stated in capsule form as "the dimensional analysis outlined below will improve the predicting of diffusion, and improve predicting more than analyses in other terms."

More exactly, the theory asserts the set of testable propositions that: Prediction of diffusion will be improved to the extent that

a. its chief factors, such as the dimensions, called "interactors," "acts," "time," "space"
b. are measured with high precision, validity, and reliability
c. and have their intercorrelations computed with each other
d. and with systematic models which hypothesize uniformities in the phenomena under specified conditions.

Our theory of diffusion will be stated in more detail in models (Refs. 6, 8, 9, 10, 20, 21) since these specify not only the variables, the constants, and their relations but also the social pre-conditions (or assumptions mathematically) and the procedures for verifying both the assumptions and eventually the whole theory. Each model specifies an hypothesis under specified conditions; the full set of consistent hypotheses which are only outlined here constitutes our complete theory of social diffusion.

Our theory is a predictive one in that the test of it is: How well will it predict diffusion in new situations in the future? The focus of attention will always be the quantitative process of diffusing, seeking to answer the question "How many individuals will exhibit the new behavior (which is at issue) at any given time?" This quantitative analysis builds on prior qualitative analysis, of course. The qualitative analysis, which is here the first stage of our dimensional analysis, seeks to classify the phenomena of diffusing into subclasses whose behavior is so homogeneous and stable relative to the predict and that the predictions about any member of one subclass will be excellent.

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II. An Analysis of Diffusion

A. Analysis into dimensional factors

Social diffusion is here defined as *the growth and spread, from person to person, of a one-way change of behavior.* The diffusing behavior, or novel act, appears at least once and cumulates in more and more people in a larger area as time goes on. This definition implies that diffusing is a product (not a sum!) of at least six factors, namely, the *actors* and *reactors,* their *acts,* the *timing, spacing,* and other factor *conditions.* Thus in the interact, which may be diffusing a rumor, the factors are the tellers and hearers, the act of telling, together with its temporal, spatial and other factor conditions such as the intensity, frequency and recency of the stimulus, etc. This product* of the six factors is a change of *interaction* of people—the one-way or transitive case (from diffusers to diffusees) of some new item of behavior.

In contrast to the factors or *dimensions* of diffusing, the remaining variables correlated with the diffusing are addends which we usually call the additive or alternative *conditions* of diffusing. All the conditions, whether factors or addends, may be usefully subclassified by tense of time as:

1. pre-conditions—including all causes;
2. co-conditions—including all concurrent variables if correlated to the diffusing;
3. post-conditions—including all consequences.

The theory outlined here does not discuss the past and future conditions of diffusing phenomena, and is an incomplete presentation to that extent. For we see a complete theory as systematizing operationally defined variables related together in all three tenses. Since this diffusion theory is a dimensional one, it explores first how large a part of diffusing can be explained and predicted from knowledge of the six basic factors, or dimensions, in the present tense, before going on to explore what residual past or future conditions one also needs to know in order to improve predicting the diffusion still more exactly.

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* This product takes care of some of the old objection that "a whole is more than the sum of its parts." The logical, mathematical, and social test of a variable being a factor is whether the product vanishes when the factor is absent, i.e., becomes zero in amount. Thus there can be no diffusion if there is no diffuser, or diffusee, or no time involved, etc. The empirical test of this "factor hypotheses" is whether the multiple correlation of the logs of the variables will predict observed diffusing better than the raw variables (It has regularly done so in the situations tested to date.)

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1. Acts, A. The first factor in any diffusing is an act, some novel behavior, whether telling a new rumor, reading a news item, buying a new product, following a new fashion, adapting imitatively to any new situation, educating children, acculturating immigrants, or any other subclass of new acts. We study and test all-or-none acts in populations of 100 to 1,000 persons at first, for simplicity. Later, the theory can be readily extended to include acts in degrees, or in series, or in combinations of any desired complexity. Often the acts and reacts of actors and their reactors need to be distinguished, as when a telling of a rumor may not be equivalent to a hearing of it.

The most basic compound unit with the act as a factor is the "potency" of diffusing. It is the per capita activity defined as the acts-per-actor-ratio. $[AP^{-1}]$ In communicational diffusion, wherever one act converts the non-knower to being a knower, the potency is the hearers-per-teller ratio. If this ratio is high it measures a potent telling or diffusing act which will tend to spread far, fast, and fully. A potency above unity means a waxing diffusion; a potency below unity means a waning diffusion; a potency of 1 means "maintenance" diffusion.

2. Actors and reactors, P_I, P_{II}. A second factor in the diffusion product is the number of people, P, involved, either as diffusers or diffusees. This population factor could appear at any of four levels according as its power or exponent is 0, 1, 2, or 3.* Thus $[P^0]$ denotes a person; $[P^1]$ denotes a plurel, a sum of persons, the actors; $[P^2]$ denotes a group, a product of acting and reacting persons, i.e., interactors; and $[P^3]$ denotes an organization, a power or triple product of acting, reacting, and role-acting persons. (See Ref. 7) Each of these levels will modify the diffusing process. Thus we have found that exponential curves predict well the diffusing from mass media in plurels, while logistic curves predict better the diffusing within a group—just as their two dimensional formulas expect. In this paper we sketch only sample cases of diffusing in a homogeneous or

* The square brackets denote dimensional formulas which relate classes of magnitudes together irrespective of the statistical indices, units, or origin points in terms of which the magnitudes may be expressed. They thus transcend statistical minutiae in order to interrelate highly abstract concepts with rigor. The necessary further specifying of indices, units, and origins which converts dimensional formulas into statistical formulas is done respectively by the three corner scripts other than the exponent in our S-notation, namely: origins $= s_{\times}$

units $= s \quad s =$ indices

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unorganized group, the [P^2] case, where the act spreads from person to person. Such diffusing may be constrained by various forms of group structure of which sample structures (e.g. "iadic," partly "iadic," and "ramified") are sketched below.

3. Time, T. The time dimension in diffusion appears in different forms depending on the origins, units and operations one uses. Using fixed origins yields *dates* in "public time"; while individual origins yield *ages* in "private time"; and using any current moment as zero point yields the past, present and future *tenses* of time.

Using clock or cardinal time units yields durations. Sequences, or ordinal units of time, measure repetitions, removes of retellings from "first-hand" to "n-th hand", as well as all familiar earlier-later sequences as in the concepts of "cause and effect," "stimulus and response," "means and ends," "action and reaction," etc.

Ordinal time units may be highly useful for transcending daily and other cycles that often obscure diffusion trends when stated in cardinal time units. Our experiments using a "random net" model and ordinal units can predict the time course and the outcome of diffusion better than we can do as yet with cardinal units.

These different manifestations of time dimension when multiplied by the activity dimension yield "periodicity," or statistical time series, which are distributions of the acts in time. Some of the diffusion models involving "rectangular," "concurrent," or "serial" periodicity will be sketched below.

4. Space L. Another dimension of diffusion phenomena is space, subclassified into dimensional "levels" as points, distances or lengths, areas, and volumes. In diffusing, space acts as resistance, requiring more effort from diffusers to overcome it the longer the intervening distance to diffusees. Consequently, distance appears in the denominator or with a negative exponent [L^{-1}]. Our studies and those of J. Q. Stewart, G. K. Zipf, and others tend to show, both from empirical data and from geometric deductions, that spatial diffusion decreases harmonically, i.e., with the first power of the distance traversed. We have twenty-seven sets of diverse data (Ref. 1) in which the mean fitted exponent on the distance factor was minus unity. The space factor in diffusing will not be sketched further in this paper, however.

5. Factor conditions, C^c_c. Residual factors—which are themselves a product of many subfactors—are those of the co-conditions which behave

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multiplicatively and not additively. This class of factors includes all the residual constraints on diffusing (other than constraints expressible in terms of acts and actors, time and space). This residual class may include any relevant variables whatever, whether attitudinal or ecological, whether cultural or physical, whether semantic or methodological or teleological. This is the huge field for research, always beckoning, whose size is measured by the multiple coefficient of non-determination ($k^2 = 1 - r^2$), remaining after the previously discussed factors have been used as predictors. Again, we will omit further discussion of the many co-conditions of diffusing in this paper, since we have chosen to study the basic social dimensions of time, space, people, and their activity first.

The factors of diffusion above may now be brought together in a summarizing formula, which in turn is but a special case of our still more comprehensive dimensional theory.

Our diffusion theory describes and predicts the forms and amounts and relations of "new actors and reactors acting and reacting, under specified temporal, spatial, and other conditions." In dimensional symbols, diffusing is but a special new or one-way case of human interacting:

$$[I_{pp} = A^2\ P^2\ T^{-2}\ L^{-1}\ C^0] \qquad \text{Eq. 1}$$

This equation defines the interactance, I_{pp}, as a product of powers of at least this set of factors (A = acts, P = people, T = time, L = length, C = any set of factor conditions). The interactance hypothesis, of which diffusion is a special case, expects the interactance thus defined to correlate highly with, and so to predict, the observed interacting or diffusing. This hypothesis will be confirmed in any situation in which the closeness of fit index of correlation between expected and observed data exceeds .9 in uncumulated data (and exceeds .99 in cumulated data), provided that the goodness of fit or reliability from sampling is acceptable at the 5% confidence level.

The interactance may be expressed in "unit form" as "one actor stimulates by unit act one reactor, to unit response, in unit time, across unit distance, under unit conditions." (This means that standardizing the units, such as in our potency ratio, is a prerequisite to research.) The interactance may be paraphrased further in interrogative form as: Who?, does what?, to whom?, where?, when?, how?, and why? More specifically for diffusion, the formula should help to answer such questions as: How fully through the population will the novel behavior spread? How far? How fast? How faultily? Under what facilitating conditions? etc.

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This diffusion formula, Eq. 1, in dimensional form, expects, as hypotheses:

a. this set of factors to predict diffusing better than other sets;
b. their multiplicative combination to predict best;
c. their weighting by exponents to improve prediction further;
d. the statistical indices, units and origins (which are specified by scripts at the three remaining corners of each base letter in our standardized matrix script notation and which are not specified in a dimensional formula) to be further operationally defined in any given research.

This diffusion formula, when in statistical form with exponents and other parameters stated in numbers, should predict the amount as well as the form of the course and outcome of a given diffusing process from knowledge of its start or preconditions. Thus, given the starting potency, and the experimental observation that the potency waned harmonically with removes, the random net model, developed by Professor Anatol Rapoport, has successfully fitted($r = .99$ in uncumulated data) the waning course and final proportion of the population reached in a case of diffusion (which was measured while the diffusion was still waxing).

B. *Analysis of the timing and grouping factors.**

There are many kinds of conditions expressible in mathematical parameters available, from which a choice can be made for priority of study in order to develop a consistent set of models for a theory of diffusion. Out of the total set available we have chosen to illustrate here some *timing* and some *grouping* factors ($[T^{-2}]$ and $[P^2]$ dimensionally): These are (1) the manner in which any individual's contacts with his acquaintances vary with periodicity in public time; (2) the extent to which an individual experiences repetitious contacts with acquaintances, varying in ordinal time units from a private time origin; and (3) the acquaintance structure (i.e., interrelations or grouping by some criterion) within any given population.

* This section on timing and grouping factors develops special cases of the general dimensional theory of diffusion. Both general and special parts of our diffusion theory are only sketched in this paper and more fully developed elsewhere. See Refs. 1, 2, for Dodd's dimensional analysis and Ref. 4 for his interactance hypothesis which subsumes diffusion. See Ref. 20 for Winthrop's extensive treatment of his eighteen basic diffusion models dealing with periodicity, repetitions, and group structures. The treatment of these eighteen cases constitutes a minor part of the mathematical manuscript of 370 pages which may be obtained for research purposes in mimeographed form from the Washington Public Opinion Laboratory.

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With respect to the first of these, we assume that there may be a *periodicity* about the number of contacts an individual makes with his acquaintances, a periodicity which may be governed by such considerations as the hourly rhythms of the work day, calendar week, or any other socially imposed artifact. Periodicity in turn may be classified into three types: rectangular, concurrent or serial periodicity. (See Figure 1.) In rectangular periodicity (which is really a-periodicity) the contacts any individual makes per unit time are constant. In concurrent periodicity the number of contacts any person makes over time changes from period to period, and this number is uniform for all members of any given population, at any given time. The cycle is constant and concurrent for the group, but each person enters the cycle at his own date. In serial periodicity every individual also possesses a variable number of contacts with time, but unlike the situation with concurrent periodicity, not all individuals at any given date are at the same phase of their cycle. Thus every time an individual is converted to new or novel behavior he starts his own conversion cycle at its initial phase, although the date at which such converting takes place may have any value whatsoever.

Visually these three types of periodicity may be presented in the following fashion:

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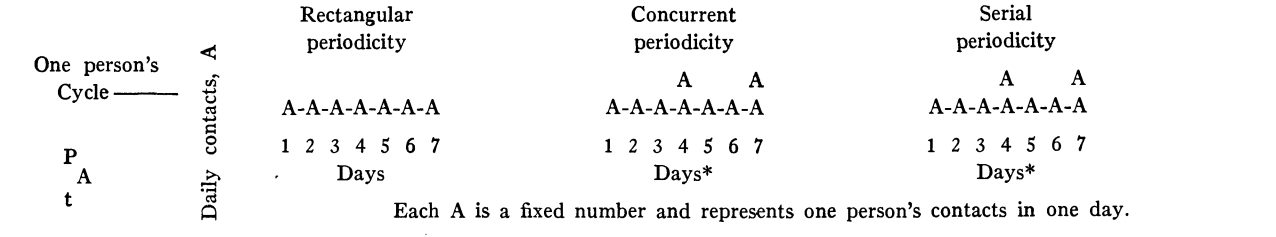
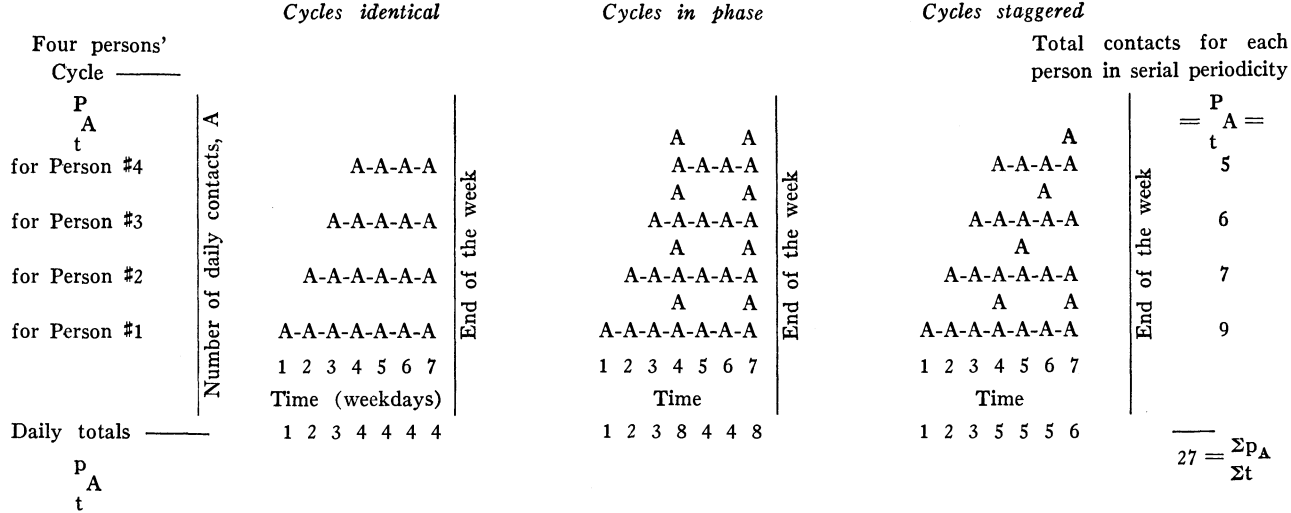
FIGURE 1
PERIODICITIES

Three types of a person's daily activity in a weekly cycle are shown.
Activity, ${}_tA$, is the number of acts by one person in each day and is also the number of new daily converts (if one act converts).
Cycles of activity are the same for all persons though differing in phase. Each person's cycle starts when he is converted but all cycles end together.



Each A is a fixed number and represents one person's contacts in one day.



* The sample cycle shown here shows that two sets of persons met on Wednesday and Saturday and one set on each other day of the week.

Formulas in script notation: A = an act

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Turning from periodicity, consider next another aspect of the time dimension which appears in the form of repetitions of the interact.

Repetitions are basically of two kinds. These are (1) repetitions with one's own converts, either in the same or in different time periods, and (2) repetitions with other men's converts, either in the same or at different time units. Most generally, both of these types of repetition take place in diffusion. The exact specifying of some of the possibilities of repetition is illustrated in the matrices of Fig. 2.

In Fig. 2A the actors (such as tellers of a rumor) are listed as row headings and the same population is again listed as actees (such as hearers) in each column heading. Each cell entry in this repetition net or sociomatrix records the number of times the row person told the message to the column person. The overtellings as well as "first tellings" are thus recorded here. The arrays are arranged to make successive sections of the matrix show the details of diffusing in successive removes or retellings. Each section, or band of arrays, represents a generation of interactors from first hand hearers at the left and top to nth hand hearers at the right and bottom. The characteristic pattern of this repetition net, arranged by generations, is that all cell entries below the main diagonal are zeros and the non-zero cell entries above the diagonal are "stepped", going "downstairs" to the right.

This "step" pattern is even more visible in the chain net, Fig. 2B. Here each teller tells only one person ever, so all the cell entries occur in one diagonal array (just above the main diagonal representing self-relations) whenever the actors or arrays are arranged in order of their removes from the starters. This Fig. 2B shows a ramified structure with unit activity rate or "unit potency ever." This means each teller tells one and only one hearer. This forms a geometric series in which the rate of growth is exactly unity so each generation just maintains its size in the next generation. The cell entries in Fig. 2B are shown recording the dates when tellers told hearers. The cell entries may show other nets, of course, such as when they record the amount of the interact between the two row and column persons, or show any other pair-property in each cell.

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FIGURE 2

MATRICES OF SEQUENTIAL INTERACTING

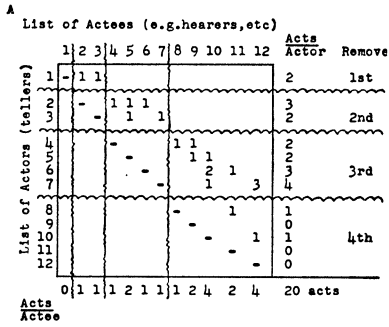

Fig. 2A *A Repetition and Generation Net*
measuring multiple tellings and hearings by generations or removes
cell entry—number of tellings by one teller to one hearer
Row sum = number of tellings by one teller
Column sum = number of tellings to one hearer
Sections (e.g. sets of arrays) = generations on removes in sequence
Note the pattern:
Entries in upper right triangle only
Entries in main diagonal sections denote intra-generation acts
All other entries must be in diagonal sections just above the main diagonal.

Matrix formula $_{u}^{t:p:q}A$

which asserts at each of t removes, each of p persons acts on a subset of q persons u times. (p, q, and u are variables)

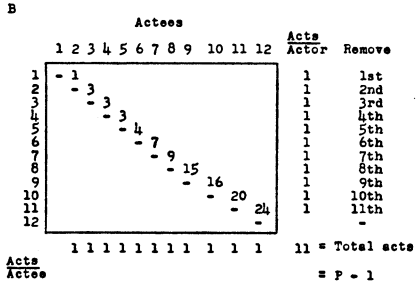

Fig. 2B *A Chain Net (with Dates)*
a ramified structure
showing "unit potency ever" i.e., one hearer-per-teller only—with the date (day of month) as entries here.
This is a geometric series with a growth rate of unity.
Cell entries may be used to record:
a) amounts of the interact
b) the date of the interact (as here)
c) repetitions
d) an attribute of the interact or of the pair of interactors, etc.
Note pattern of diagonal entries just above main diagonal
Matrix formula = $P:Q:R \ldots :Z_A$
which asserts "a person acts on another person who acts on another person, etc., to the Z-th person."

Next, in sub-classifying a group by its structure, one must always specify the criterion of structure, that is, the particular variable that is observed and entered in the cells of the matrix in Figure 3 and which records the ties of one kind between the members of the group in all possible pairs. The pattern of these ties, the filled out matrix, specifies the structure of the group *by criterion* C_1 (or C_2, C_3, etc.). For purposes

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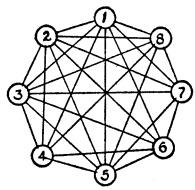
FIGURE 3

SOME GROUP STRUCTURES

In diagram form *In matrix form*

"Iadic" structure—fully interrelated

A



A group of persons all tied together (as shown by connecting lines)

$$\frac{\text{Possible ties}}{\text{Actual ties}} = \text{index of "iadicity"} = 1.0 \text{ here}$$

B

List of actees, e.g., known persons, etc.

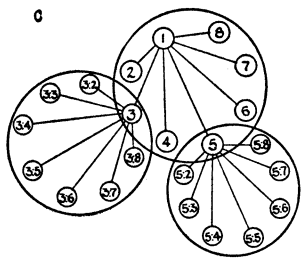
List of actors, e.g., knowers, etc.

	1	2	3	4	5	6	7	8
1	-	1	1	1	1	1	1	1
2	1	-	1	1	1	1	1	1
3	1	1	-	1	1	1	1	1
4	1	1	1	-	1	1	1	1
5	1	1	1	1	-	1	1	1
6	1	1	1	1	1	-	1	1
7	1	1	1	1	1	1	-	1
8	1	1	1	1	1	1	1	-

A 1 in a cell means the row person acts on the column person; a 0 in a cell means he does not act on the column person. Matrix formula = $p:pI$ which asserts "for every pair of persons there is an index of one way interrelation, I".

Ramified Structure—branchingly interrelated

C



None of an actor's actees act on each other, e.g., all of each person's friends are strangers to each other.

D

List of actees

List of actors

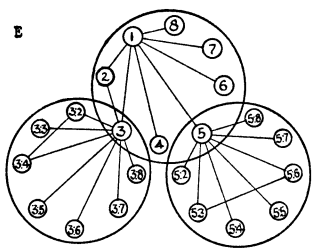
	1	2	3	5 5:2	
1	-	1	1	0 0 0	0 0
2	0	-	0	0 0 0	0 0
3	0	0	-	1 1 1	0 0
3:2	0	0	0	- 0 0	0 0
3:3	0	0	0	0 - 0	0 0
3:4	0	0	0	0 0 -	0 0
5	0	0	0	0 0 0	- 1
5:2	0	0	0	0 0 0	0 -

denoting omitted arrays

The same branching structure arrayed in a matrix for systematic and mathematical treatment. Note pattern of "steps" in upper triangle. Matrix formula = $p:qI$ which asserts "for every person as actor there is a different set of actees, (including any zero set)."

Partially Ramified and Partially Iadic structure—partly interrelated

E



A branching structure overlaid with incomplete tying of the members in each circle or branch.

F

List of actees

List of actors

	1	2	3	5 5:2	
1	-	1	1	0 0 0	1 0
2	1	-	1	0 0 0	0 0
3:2	1	1	-	1 1 1	0 0
3:3	0	0	1	- 0 1	0 0
3:4	0	0	1	0 - 1	0 0
5	0	0	1	1 1 -	0 0
5:2	1	0	0	0 0 0	- 1
	0	0	0	0 0 0	1 -

denoting omitted arrays

The same structure arrayed in a matrix, cross-classifying all possible members of any grouping studied. Matrix formula $p:qI$ which asserts "each person is interrelated to his own subset, q, of any other persons."

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of predicting diffusion, relevant criteria may be such variables as (1) acquaintance (e.g., "List your friends here"); or (2) frequency of contact (e.g., "Whom did you talk with yesterday?"); or (3) congeniality of interest (e.g., "Who do you think would like to do, or share, this sort of thing with you?"); etc. In trying to predict diffusion from group structure or sociometric networks, it is necessary that the criteria be independently observable before the diffusing and that the criteria act with full constraint upon the diffusing. Full constraint means that a criterion network (i.e., the matrix entries) are perfectly correlated with the diffusion network. Thus for example, diffusing would go *only* through acquaintances, or *only* through frequent contacters, etc. Of course, in practice this perfect correlation which is assumed by a model does not hold. This, however, is precisely the problem of modeling—to find out experimentally to what degree such assumptions hold and how the model must be revised to reduce discrepancies between its expectations and the observations (if made under appropriately controlled conditions).

The structure of a group by any criterion is sub-classifiable in many way, of which only three types will be noted here. Population structures will be called *completely "iadic," completely "ramified,"* or *partially both.* When each individual of a population possesses a circle of acquaintances no two of whom know one another, the population is said to be completely ramified by virtue of the fact that if, in such a population, each individual contacts and converts a certain number of other individuals per unit time to the novel behavior, the diagram of the conversion or diffusion process over time would show a branching or ramified structure, as in Figure 3, C and D. On the other hand, in a population of i persons all of whom, for one criterion, knew each of the remaining i-1 persons, one might speak of the plane polygon which would represent this acquaintance structure as an i-ad, and one might speak of the population which possesses this property of all-round, mutual acquaintanceship as being completely iadic in nature, as in Figure 3, A and B. Between these extremes are populations in which every individual possesses a circle of acquaintanceship some of whose members know one another and some do not. Such a population would be partially iadic and partially ramified in nature, as shown in Figure 3, E and F. The degree to which any population is iadic is in part a function of the number of pairs of one's acquaintances who are likewise acquainted with each other. This degree of amicability of a population is referred to as the iadicity of the population and may be measured by an index of iadicity.

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This index in general is the ratio of the actual ties (i.e., non-zero cell entries in the matrix) to all possible ties in the group of a person's acquaintances including himself.

By combining our parameters for periodicity and for group structure with the dichotomous possibility that an individual (1) may never meet converts (either his own or other people's) over the course of time or (2) will meet converts (either his own or other people's) over the course of time, there results what may be called the 18 basic cases for the timing and grouping aspects of diffusion. These cases spell out some of the possible models in the dimensional classes of "celerations" and of "groups" defined by the formulas $[T^{-2}]$ and $[P^2]$.

These eighteen cases may be tabled here to suggest the present degree of development of this theory of social diffusion. The mathematical exposition of these eighteen types has been developed by the junior author in a 370 page manuscript (Reference 21). Only one type, "Case 11," can be briefly sketched here with two preliminary experiments exploring its applicability.

TABLE 1

CASES OF DIFFUSION CLASSIFIED BY PERIODICITY, REPETITION, AND GROUP STRUCTURE

Structure	Periodicity (No repetitions)		
	Rectangular	Serial	Concurrent
Ramified	Case 1, 10*	Case 4, 13*	Case 7, 16*
Iadic	Case 2, 11*	Case 5, 14*	Case 8, 17*
Mixed	Case 3, 12*	Case 6, 15*	Case 9, 18*

Repetition is allowed in the starred cases (10 to 18) but not in the others. These cases constitute a *first approximation.* By then introducing any number of other parameters into any one of these structures, whether the parameters are psychological, sociological, economic, neurological, etc., one can alter the mathematical models obtained in the first approximation so as to have them conform more and more closely to any degree of socio-psychological reality in which one may be interested.*

* The reader who is interested in the mathematical analysis of these 18 cases or in the mathematical analysis of various socio-psychological aspects of diffusion not dealt with in these eighteen cases, may obtain copies of these by writing to the Washington Public Opinion Laboratory.

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C. *Methodological tools for analysis*

There are at least three different but complementary approaches or symbolic devices for representing the growth of new behavior in any given population. These are (1) analysis in terms of the infinitesimal calculus, (2) analysis in finite difference terms which, in turn, divides into analysis in terms of combinatorial algebra and analysis in terms of increment-schemas, and (3) analysis in terms of matrices. Each of these will be described briefly, since the operations they specify are part of the operational definition of this theory of diffusion.

1. Finite differences and infinitesimal calculus

All growth of novel social behavior will be essentially of one of two types of distribution in time. Either the growth of converts to new behavior will be continuous streaming process in time or will exhibit a regular or irregular cyclicity, that is to say, will show "bunching" at certain points in time with negligible growth between these points of "bunching." Where the growth of such behavior is "bunched," finite difference techniques applied to the analytic formulation of theoretical models for the study of social diffusion may be appropriate. "Bunching" may also arise from regular fluctuations which occur within every time interval of growth or from different types of fluctuations within each interval, all of which, however, may be accounted for by some common rationale. If continuous streaming is present, the use of the infinitesimal calculus and probability theory is called for. In any event, discrete (discontinuous) formulations of growth in finite difference terms are *nearly always* directly translatable into continuous formulations and models especially for large populations, etc. At present we believe the calculus may be the better tool for populations which are large and whose relevant structuring in detail is unknown; while finite differences may be the better tool for small groups whose relevant structuring can be measured. Further experimentation is needed to chart the conditions under which either the finite or infinitesimal difference models are likely to predict diffusion best under given conditons. In this paper illustrations of diffusion technics and experiments will be limited to the finite differences technic which the junior author has developed extensively.

2. Increment schemas

A second semantic tool for studying diffusion is the increment schema. This schema tabulates the converts by their converters and by their generations or removes from the start. It cross-classifies the knowers or converts

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to the novel behavior by their ancestry and by their generation in the series schema, i.e., it subclassifies converts along the population and time dimensions.

An increment schema arrays the diffusion data by "generations" in successive rows which widen out with each generation. This schema forms a triangle whose base expands with each new generation. Thus in the first row, representing the zero-th period (or apex of the triangle), write the number of starters of the diffusing process. In the second row, representing the first period of diffusing, write the increment or number of converts made by the starters during the first period. In the third row, representing the second period (or "generation" or "remove") of diffusing, write the two increments who were converted by the starters or zero generation persons plus those converted by the first generation increment which was written in the second row. In the fourth row, write similarly the sum of the increments produced by all the increments in the preceding rows or generations. Continue this process of unrolling the schema to n-generations. Thus, however regular or irregular the rate of converting and the consequent increments may be, the schema provides a detailed record. With suitable subscripts, as in combinatorial algebra or as in matrices, each increment can be tagged to show its generation and all its ancestry. If the increments are analyzable into regular patterns (as when, for example, each generation doubles itself), combinatorial formulas in this increment schema can give the sum of the series, the sum in any one generation, the descendants of any one ancestor, the ancestors of any descendant, etc. If a complete record of ancestry and descendants is desired for each individual in the population and not simply for each increment of population, then the interperson matrix must be used as shown in Figure 3 B, D, and F.

3. The matrix as an analytic tool.

Among the finite difference technics another powerful tool for exact analysis of diffusing or of any interacting is the matrix. A matrix tabulates items in rows and columns, making one orderly array for each variable. See Figure 3 for examples. Each cell contains an entry which, whether a number or a quality, is a logical product of its row and column (and sagittal array if in a "solid" matrix with three axes). The cell entry is jointly characterized by every array or variable in which it occurs. Thus in Figure 3, the population is listed *as actors,* each person heading one row and is again listed in the same order *as actees,* each person heading one column. The cell entry states or measures the interrelation of the row

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actor and the column actee thus specifying which persons as diffusers contacted which persons as diffusees.

The 2-matrix with two axes as shown on a "page" in Figure 3 may be expanded to an n-matrix with n axes to deal with n variables *simultaneously* in any social situation studied. Thus the 2 matrix cross-classifying actors and their actees are shown one "page" as of one date, and may be expanded along the time axis by superposing another "page" for each added date. This makes a "book" or 3-matrix cross-classifying the two dimensions of group structure or interactivity with the time variable. The increment schema above is a "slice" through such a "book" as it is a 2-matrix cross-classifying people with generations of time. Thus all dynamic phenomena of groups can be ordered for exactly specified and rigorous study of social processes, diffusing, change, ages, sequences, dates, tenses, or any other temporal aspect of activity.

An alternative to expanding a matrix along another perpendicular axis is to expand it with further "sections" along the existing axes. Thus each successive period of time would add its matrix section (i.e., sets of rows and columns) as in Figure 2. When the successive sections represent successive generations or removes, the matrix spells out the descendants and ancestors of everyone in a fully specified increment schema.

With one added axis it can cope with any number of diverse kinds of behavior by providing a "shelf" of "books" for each kind of behavior. With another added axis it can cope with spatial series, recordable in "bookcases" of "shelves." By adding an axis for every variable studied which cross-classifies all the other variables the detailed structure and functioning of group activity can be specified and treated mathematically. In short, the matrix orders the phenomena of groups preparatory to any further analyses or syntheses the researcher may desire. Thus the matrix provides a place to record comprehensively yet precisely all possible human interrelations.

These and many more uses of the socio-matrix for law-discovering and law-generating studies of strictly social or *inter*human phenomena are only beginning to be appreciated by sociologists.[1]

[1] See the bibliography attached and fuller listing in Reference 7 for the growing use of sociomatrices.

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III. Two Experimental Tests of Diffusion Models

The testing of a finite difference model in this diffusion theory may be illustrated by two experiments.[2] These were designed to test a logistic or continuous curve model and a geometric series model so that applying Winthrop's "Case 11" model (which was formulated independently of these data) will be a more rigorous test of such a model. For one of the social conditions (the hearers-per-teller ratio) was not known in advance and had to be guessed in order to apply the mathematical assumptions of the model. Therefore a very close fit is not to be expected until more definitive tests can be made. However, these tests illustrate the methods of testing the theory and the high degree of fit resulting from even this imperfect testing.

The model applied here is "Case 11" among the eighteen basic models which vary periodicity, repetition, and group structure as listed in Table 1. This model is deduced from the three assumptions of rectangular periodicity, no repetitions, and completely iadic structure. Alternatively stated, the model deals with a social situation where the diffusing is at a steady rate, new, and equally possible for all the population. Note the implications of each of these three mathematical assumptions or social preconditions.

Rectangular periodicity here means that everyone, on the average, retells the message steadily from period to period (or from remove to remove, if ordinary time units are studied). The variance of the potency, or mean hearers-per-teller ratio, will be small over the set of equal and consecutive periods studied. Obviously this is a statistical approximation in which the question is: Are the actual deviations from steady potency large enough to result in a poor fit and lower the prediction?

Complete iadicity, or interrelatedness, here means that everyone is acquainted with everyone else at least well enough to have equal opportunity to retell the message. The group structure is "homogeneous" with respect to the probability of retelling. The probability of anyone telling anyone is equal for all. Of course, as the group gets larger its members will not all be acquainted. But we believe, from preliminary experiments not yet explicitly checked, that this condition of equal opportunity to interact is approximated in a large population if each person's circles of acquaintances

[2] These were two of a score of "pretests" made during the first or exploratory year of Project Revere. This project, conducted by the Washington Public Opinion Laboratory on contract with the Human Resources Research Institute of the Air Force studies how to maximize the social diffusion of a message started by leaflets dropped from planes.

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for various purposes overlap with other person's circles in a sufficiently random manner as yet unspecified. This will tend to make the whole community in effect interconnected even though each person is acquainted with only a few and variable number of other persons. Iadicity means that there are no physical or cultural barriers which are correlated to the diffusing and therefore nothing will operate to lessen the diffusing and make opportunities to interact unequal in effect. The population may be very heterogeneous in a thousand respects provided each respect is uncorrelated with the diffusing activity. In this sense the population should be homogeneous-relative-to-the-diffusing or show an equal chance for everyone to interact with a few people around him.

The third condition of allowing no repetitions means that only first hearers, or newly contacted persons, are considered. All repeated contacts of a person already converted, i.e., one who is already a knower of the message, are neglected.

These three pre-conditions of *steady, new* and *equal* interacting are substantially the conditions giving rise to a logistic S-shaped continuous growth curve of which Case 11 is a finite difference form.[1]

Pretest Q, was a diffusion experiment in a Y.M.C.A. summer camp in July 1952.[2] A novel behavior was spread to 37 out of 42 boys during the two hour rest period and its growth recorded in the number of converts made every fifteen minutes. Three "starter" boys (a leader, an isolate, and a "middler" on a sociometric test given earlier) were given a large yellow pin with the name of the camp and a question mark in black on it. Each was told:

> "Here is a special pin. It is important that you wear it today. We have a few more of them, but not enough for all the campers. Those campers who find out about this and come to the 'Old Lodge' will also get a pin, as long as our supply lasts. Remember, it is important that you wear it today, and remember that we don't have very many pins."

As each boy went back into the camp, others saw the pin, asked about it and came to get one like it, diffusing the pin-wearing behavior through the whole camp. Note that the distance factor between interactors was negligible here; the stimulus and other conditions were kept fairly constant;

[1] The detailed mathematical derivations, discussion and computation of these models here may be obtained on application from the Washington Public Opinion Laboratory.

[2] The experimental design and fuller report on Pretest Q by Richard J. Hill and Otto N. Larson and on Pretest M below by Arch Cooper may be obtained on request.

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the group was small and all knew each other; the time was short with no cyclic or other patterning from competing activities; and the act of acquiring the pin was unrepeatable. According to the formula* for Case 11 the expected and observed number of converts in each quarter hour as shown in Table 1 fits fairly well, though not up to our standard of a closeness of fit correlation coefficient over .9 on uncumulated data. The closeness correlation was .85 (which was acceptable at the 5% confidence level). We conclude that this model shows promise under these conditions and should be tested further with better data. (Two further sets of campers are being tested.)

TABLE 2

GROWTH OF THE PIN-REQUESTING BEHAVIOR AT A CAMP (A TEST OF A DIFFUSION MODEL)

Clock time interval t	Expected converts e	Observed converts o
12:45	3.00	3
12:46 to 1:00	2.76	0
1:01 to 1:15	4.84	6
1:16 to 1:30	7.74	9
1:31 to 1:45	9.17	8
1:46 to 2:00	7.15	5
2:01 to 2:15	2.08	3
2:16 to 2:30	0.37	2
2:31 to 2:45	—	1
Total	37.11	37

r_{eo} = .85 on uncumulated data showing fair fit of the Case 11 model expectation to the observed data.

A second experiment provided a test of a variant of Case 11. Pretest M, which was called Chain Tags, was repeated in each of four matched

* The formula for Case 11 is: $_{T}P = {}^{T-1}P\ A/_{PT}\ (1 - {}^{T-1}P/P_D)$
where $_{T}P$ = the increment in converts during period $_{T}1$
^{T-1}P = the converts up to the end of the previous period
$A/_{PT}$ = the potency rate or acts-per-person-per period the diffusing activity index (here guessed at 1.0 for lack of advance data) later found to be .92
P_D = the total diffusable population (here 37 boys)

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villages of about a thousand population each. The chain letter idea was modified to get people to pass on a printed message from person to person. Each of the ten representative starters in each community was handed a cellophane wrapped packet of return addressed postcards all alike. Each packet broke down into a "tracer" postcard to be filled out and mailed in and a subpacket to be given to someone else. Each packet contained a series of ten subpackets allowing it to go to a chain of ten persons at most through ten removes, each tagged by its tracer postcard. Each card recorded the name and address of the respondent and his preceding and succeeding "links" with the time, place, etc. of getting the packet. The motivations to cooperate were four, one in each town, namely "to help science" in the first town, "to help Civil Defense" in the second town, "to help a modern Paul Revere in an emergency" (i.e., to test whether a vital message could be spread in this person-to-person way) in the third town, and "to earn a dollar" (a payment for responding) in the fourth town. By using remove units of time and by allowing three weeks for the diffusing to finish (before the deadline printed on the cards) the effect of daily and other cyclic activity was eliminated. Distance may have been a factor though we believe it correlated very little with the diffusing (the address data was not adequate to compute the r). The stimulus, motivation, and other conditions seemed fairly constant within each town. The upper limits of the starting population, the diffusable final population, the activity rate of potency, and the overall period were all experimentally controlled in this Chain Tags test.

The formula* used here was a variant of Case 11 developed by Winthrop so as to match the social condition that only the last remove persons (not the cumulated total of recipients) were able to pass on the message to the next remove persons. The agreement for the four towns combined between the model's expectation and the observed data is shown in Table 3.

The closeness of fit correlation for the uncumulated data was .93 with a goodness of fit from sampling acceptable at the 1% confidence level.[1] The

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* The formula therefor is: ${}_{T}\mathrm{P} = {}_{T\text{-}1}\mathrm{P}\ \mathrm{A}/_{PT}\ (1 - {}^{T\text{-}1}\mathrm{P}/\mathrm{P}_{D})$
where ${}_{T\text{-}1}\mathrm{P}$ = the increment in converts in the ${}_{T\text{-}1}1$ period
and $\mathrm{A}/_{PT}$ = 1 since each receiver gets only one subpacket to pass on.

[1] The closeness of fit correlation was .999 in the cumulated data. This is a spurious correlation since cumulating compels some correlation even of chance

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TABLE 3

GROWTH OF THE MESSAGE-RECEIVING BEHAVIOR IN FOUR TOWNS

Order of the remove	Expected receivers	Observed receivers
zero-th	40.00	40
1st	28.00	28
2nd	23.00	23
3rd	21.19	23
4th	24.97	20
5th	18.88	19
6th	19.38	16
7th	14.56	10
8th	7.12	7
9th	5.36	7
10th	6.97	8
Total	209.	201

$r_{eo} = .93$

degree of agreement between the hypothesis specified by the model and the observations is well over our standard of a closeness of fit correlation exceeding .9 and the 5% confidence level. We conclude therefore that this model fitted the data well here, and under like conditions (of random and steady chain interacting) can be expected to predict diffusing well in new situations.

It must be remembered that the model called Case 11 here states one hypothesis among the many score of hypotheses that comprise our diffusion theory. It deals with the case of steady activity under equal opportunity.[2] It therefore tests only a small part of the total dimensional theory of diffusion which requires much further testing.

related items. It is noted here, however, partly for comparison with other curve fits reported in cumulative data in the sociological literature. It is also partly in order to emphasize the principle that reporting correlations in uncumulated data is the more exacting test and the only proper test for closeness of fit correlations. Thus the r from cumulated data in Pretest Q Table 2 is .9994 while the r from uncumulated data was only .85.

[2] These conditions are more operationally specified statistically by a vanishing variance of the activity rate over the time periods and non-vanishing cell entries in the matrix of group structure. The dimensional formula for this case $[T^{-2}\ P^2\ A^1]$ shows that it involves only the time factor in celerated form $[T^{-2}]$, the population factor in group form $[P^2]$, and a simple activity [A] from the set of factors in Eq. 1.

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